Homework 5
MATH 348
due at 5pm on Tuesday, October 22, 2024

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.

For this homework, you must type your solutions to all of the problems in LATEX. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 5 assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (4 points) Consider all of the possible topologies on the two-point set $X = \{a, b\}$. Indicate which topologies are homeomorphic.

2. (5 points) Exercise 5.1 in the text

3. (5 points) Exercise 5.2 in the text

4. (5 points) Exercise 5.8 in the text

5. (6 points) Prove that if $S \times T$ is compact, then S and T are both compact.

6. (6 points) Let $f : X \to Y$ be a function. The *graph* of f is the subset of $X \times Y$ given by $G = \{(x, f(x)) \mid x \in X\}$. Show that if f is continuous and Y is Hausdorff, then G is closed in $X \times Y$.

7. (5 points) Verify that the stereographic projection $S^1 - \{(0, 1)\} \to \mathbb{R}$ of Example 5.7 is given by the formula $(x, y) \mapsto 2x/(1 - y)$.

8. (4 points) A function $f : X \to Y$ is an *embedding* if and only if $f : X \to f(X)$ is a homeomorphism from X to $f(X)$, where $f(X)$ has the subspace topology from Y. Give three examples of embeddings.