Homework 8

MATH 348

due at 5pm on Thursday, November 21, 2024

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.

For this homework, you must type your solutions to all of the problems in LATEX. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the <u>Homework 8</u> assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

- **1.** (3 points) Let K be a k-simplex. For each $j \in \{0, 1, ..., k\}$, how many j-dimensional subsimplices does K have?
- 2. (5 points) Exercise 7.2 in the text: just show that any two triangulations of the circle S^1 have the same Euler number
- **3.** (10 points) Exercise 7.3 in the text
- 4. (6 points) Exercise 7.4 in the text
- **5.** (5 points) Exercise 7.5 in the text
- **6.** (6 points) Suppose that simplical complex K is the union of two subcomplexes. That is, $K = S \cup T$, where S and T are each subsets of K that are simplical complexes. Show that

$$\chi(K) = \chi(S) + \chi(T) - \chi(S \cap T).$$

7. (5 points) Suppose that K is a simplicial complex with $\chi(K) = -n$, where n is a positive integer. What is the smallest possible number of simplices in K?