

## Homework 9

MATH 348

due at 5pm on Tuesday, December 10, 2024

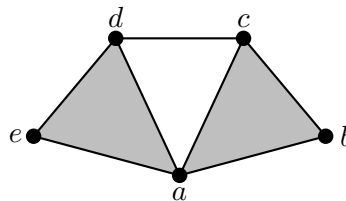
Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

*Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.*

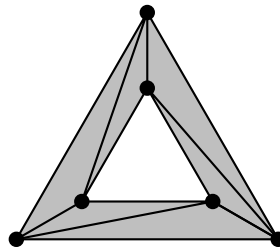
For this homework, you must type your solutions to all of the problems in L<sup>A</sup>T<sub>E</sub>X. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the [Homework 9](#) assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (8 points) Consider the following simplicial complex  $K$ .



- (a) Give an example of four 1-simplices of  $K$  that are linearly independent in the vector space  $C_1(K)$ .
  - (b) Give an examples of 1-chains (of simplices in  $K$ ) that have zero boundary.
  - (c) Give an examples of two 1-chains (of simplices in  $K$ ) that are homologous.
  - (d) For this simplicial complex, show that the composition of boundary maps  $\delta_1 \circ \delta_2$  is the zero linear transformation.
2. (6 points) Exercise 9.2 in the text
3. (6 points) Compute the Betti numbers  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  (dimensions of homology groups mod 2) of the following simplicial complex.



4. (4 points) Think of one concept that you struggled to understand or one problem that you struggled to solve in this course. Describe the struggle and how you overcame it. How was the struggle itself valuable? For instance, did the struggle build aspects of character such as endurance, self-confidence, or competence that will be helpful to you in other contexts?