

17 Sept. 2026
TOPOLOGY DAY 3

For real-valued functions with the standard topology
the type of open sets from
real analysis

THEOREM 2.9 in Crossley:

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous (by the ε - δ definition)
if and only if $f^{-1}(S)$ is open whenever $S \subset \mathbb{R}$ is open.

New way of thinking about continuous functions:
preimage of every open set is open.

EFFECTIVE THINKING PRINCIPLE (Su and Starbird)

Bring to mind all equivalent formulations [of a concept].

After learning that a concept has multiple equivalent formulations,
get in the habit of consciously considering each one in order
to select the most convenient option in a given situation.

What is a topology?

DEFINITION:

A **topological space** is a set X together with a collection $\mathcal{T}$ of subsets, called **open sets**, that satisfy four rules:

T1. The set X itself is open.

T2. The empty set $\emptyset$ is open.

T3. Arbitrary unions of open sets are open
 $\hookrightarrow$ finite or infinite

T4. Finite intersections of open sets are open.

The collection of open sets is called the **topology** on X .