

# Subspace Topology

MATH 348

1. Verify that the subspace topology on a subset  $S$  of a topological space  $T$  is, in fact, a topology.

2. Consider the following subsets of  $\mathbb{R}$  with the subspace topology.

(a) Let  $K = \{\frac{1}{n} \in \mathbb{R} \mid n \in \mathbb{Z}_+\}$ . Show that the subspace topology on  $K$  is the discrete topology.

(b) Let  $K^* = K \cup \{0\}$ . Show that the subspace topology on  $K$  is not the discrete topology.

**3.** Let  $X$  be a topological space, and let  $Y \subset X$  have the subspace topology.

(a) If  $A$  is open in  $Y$ , and  $Y$  is open in  $X$ , show that  $A$  is open in  $X$ .

(b) If  $A$  is closed in  $Y$ , and  $Y$  is closed in  $X$ , show that  $A$  is closed in  $X$ .