

Bases

MATH 348

1. The collection $\mathcal{B} = \{\{x\} \mid x \in \mathbb{R}\}$ is a basis for what topology on $\mathbb{R}$?
2. The collection $\mathcal{B} = \{[a, b) \subset \mathbb{R} \mid a < b\}$ is a basis for a topology on $\mathbb{R}$. Describe the intervals that are open in this topology.
3. Let $\mathcal{B} = \{(n, n+1) \mid n \in \mathbb{Z}\}$. Is $\mathcal{B}$ a basis for some topology on $\mathbb{R}$? If so, describe the open sets in this topology. If not, why not?
4. Let $\mathcal{B} = \{(n, n+2) \mid n \in \mathbb{Z}\}$. Is $\mathcal{B}$ a basis for some topology on $\mathbb{R}$? If so, describe the open sets in this topology. If not, why not?
5. Given a set X and a collection $\mathcal{B}$ of subsets of X , what criteria could you use to determine whether or not $\mathcal{B}$ is a basis for a topology on X ?
6. For each $n \in \mathbb{Z}$ define the set

$$B(n) = \begin{cases} \{n\} & \text{if } n \text{ is odd,} \\ \{n-1, n, n+1\} & \text{if } n \text{ is even.} \end{cases}$$

Is $\mathcal{B} = \{B(n) \mid n \in \mathbb{Z}\}$ the basis for a topology on $\mathbb{Z}$?

Give an example of a continuous function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ in this topology. Then give an example of a discontinuous function.