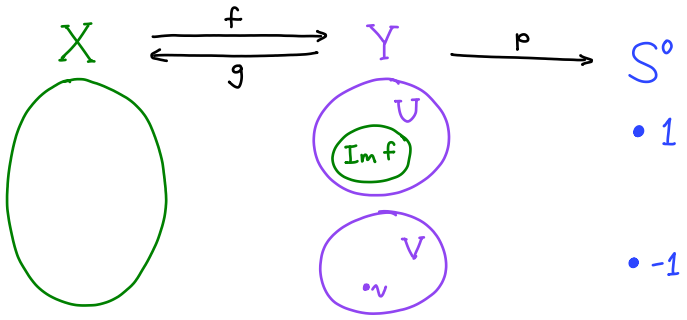
Homotopy and Connectivity

MATH 348

Theorem: If X is a connected topological space and Y is a disconnected topological space, then X and Y are not homotopy equivalent.

Use the following diagram and outline to complete a proof of the previous theorem.

Let X is a connected topological space and Y is a disconnected topological space. Suppose $X \simeq Y$. Consider the maps in the following diagram.



1. Why does there exists $F\colon Y \times I \to Y$ such that $F(y,0) = f(g(y))$ and $F(y,1) = y$?

2. Why does there exist $p : Y \to S^0$ with $p(y) = 1$ if $y \in U$ and $p(y) = -1$ if $y \in V$?

3. Define $h\colon [0,1] \to S^0$ by $h(t) = p(F(v,t))$. What can you say about h?

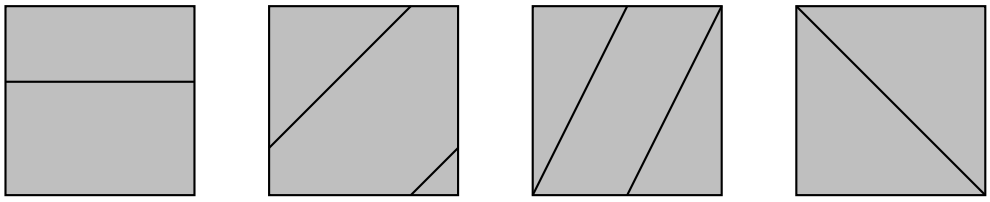
4. Why is $h(0) = 1$ and $h(1) = -1$? What does this allow you to conclude?

Now that you understand all of the steps in the proof, write the proof nicely in paragraph form on your own paper.

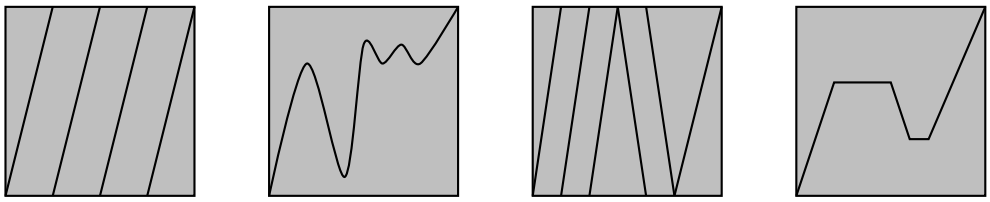
Circle Maps

MATH 348

1. What circle maps are shown in the following graphs?



2. What is the **degree** of each of the circle maps graphed above? How about each of the following?



3. Sketch the graph of a circle map with degree 5. Then sketch a circle map with degree -2.

4. Prove the statement: If a circle map $f : S^1 \to S^1$ has a nonzero degree, then it is surjective.

5. Under what condition, do you think, are two circle maps are homotopic? Why?

6. Is the circle S^1 contractible? Can you prove your answer?