

Homework 5

MATH 348

due at 11pm on Tuesday, October 27, 2026

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework. *You must include an acknowledgment of resources and collaboration as part of your homework solutions.*

For this homework, you must type your solutions to all of the problems in $\text{\LaTeX}$. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the [Homework 5](#) assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (6 points) Exercise 4.8 in the text
2. (4 points) Consider all of the possible topologies on the two-point set $X = \{a, b\}$. Indicate which topologies are homeomorphic.
3. (5 points) Exercise 5.1 in the text
4. (5 points) Exercise 5.2 in the text
5. (5 points) Exercise 5.8 in the text
6. (5 points) Verify that the stereographic projection $S^1 - \{(0, 1)\} \rightarrow \mathbb{R}$ of Example 5.7 is given by the formula $(x, y) \mapsto 2x/(1 - y)$.
7. (4 points) A function $f : X \rightarrow Y$ is an **embedding** if $f : X \rightarrow f(X)$ is a homeomorphism from X to $f(X)$, where $f(X)$ has the subspace topology from Y . Give three examples of embeddings.
8. (6 points) A **totally ordered set** is a set X with a binary relation $\leq$ such that for all a, b , and c in X the following hold:
 - (a) $a \leq a$
 - (b) $a \leq b$ or $b \leq a$
 - (c) If $a \leq b$ and $b \leq c$, then $a \leq c$.
 - (d) If $a \leq b$ and $b \leq a$, then $a = b$.

If $a \leq b$ and $a \neq b$, then we may write $a < b$.

Let X be a totally ordered set with more than one element. Consider the collection $\mathcal{B}$ of all subsets of X of the following types:

- $(a, b) = \{x \mid a < x < b\}$ for any $a < b$ in X ;
- $[a_0, b) = \{x \mid a_0 \leq x < b\}$, where a_0 is the smallest element of X , if such element exists, and $b \in X$;
- $(a, b_0] = \{x \mid a < x \leq b_0\}$, where b_0 is the largest element of X , if such element exists, and $a \in X$;

The collection $\mathcal{B}$ is the basis for a topology on X called the **order topology**. Prove that the order topology on X is Hausdorff.