Homework 7

MATH 348

due at 11pm on Tuesday, November 10, 2026

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework. *You must include an acknowledgment of resources and collaboration as part of your homework solutions.* (This is worth 2 points of this homework grade.)

For this homework, you must type your solutions to all of the problems in LaTeX. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 7 assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (5 points) A surjective map $f : X \to Y$ is a **quotient map** if each $U \subset Y$ is open in Y if and only if $f^{-1}(U)$ is open in X.

Is $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x + y$ a quotient map? Why or why not?

2. (5 points) Recall that a map $f : X \to Y$ is an **open map** if for each open set $U \subset X$, the set $f(U)$ is open in Y.

Show that a surjective, continuous, open map $f : X \to Y$ is a quotient map.

3. (6 points) Exercise 6.1 in the text. Specifically, state functions f and g that satisfy the definition of homotopy equivalences (at the beginning of Section 6.2 in the text) where $S = (0, 1)$ and $T = [0, 1]$, and show that f and g satisfy this definition.

4. (4 points) Exercise 6.2 in the text

5. (6 points) Exercise 6.3 in the text

6. (4 points) Suppose that X is a topological space and D is a disk in the plane. Show that there is only one homotopy class of continuous functions from X to D.

7. (8 points) Exercise 6.4 in the text