15 February 2024

We want to design transitions to achieve a specified steady state distribution.

① Start with some simple proposal transition

example: $Q = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

more generally, a function from a given state

② Decide whether or not to accept the proposed transition

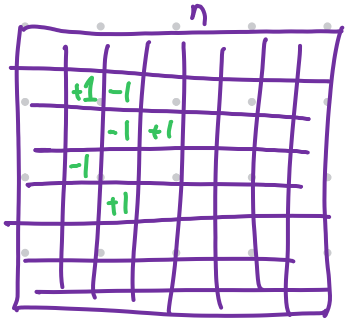
- If the proposed state has greater frequency than the current state: ACCEPT
- Otherwise, accept the proposed transition with probability $\frac{\text{weight(proposed)}}{\text{weight(current)}}$

→ this got us matrix P

TODAY: Ising Model

states are $n \times n$ grids

n



n

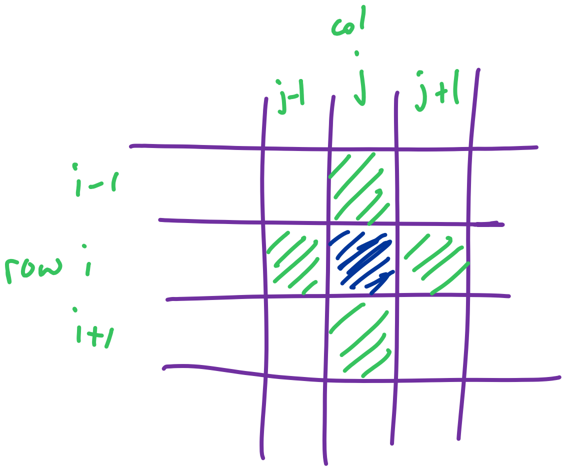
$n \times n$ grid: n^2 entries

each entry is $+1$ or -1

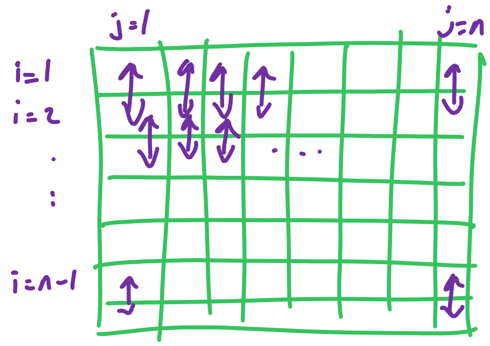
So there are 2^{n^2} possible grids (states)

e.g. if $n = 10$: there are 2^{100} states

$2^{100} \approx 10^{30}$



define $f(g)$ (g = grid) to be the number of pairs with the same value



Define $\text{weight}(g) = e^{\beta \cdot f(g)}$ for some parameter $\beta \geq 0$

Acceptance ratio: $\dfrac{\text{weight}(\text{prop})}{\text{weight}(\text{curr})} = \dfrac{e^{\beta \cdot f(\text{prop})}}{e^{\beta \cdot f(\text{curr})}}$

$= e^{\beta(f(\text{prop}) - f(\text{curr}))}$ (Δf)

← either −4, −2, 0, 2, or 4