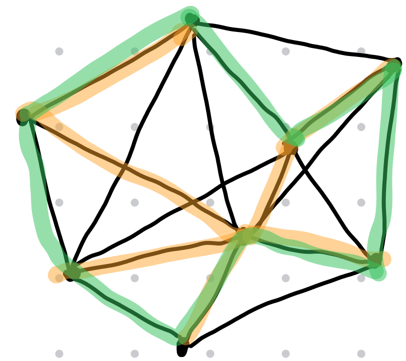
21 March 2024



Min Spanning Tree (MST)

TSP Tour

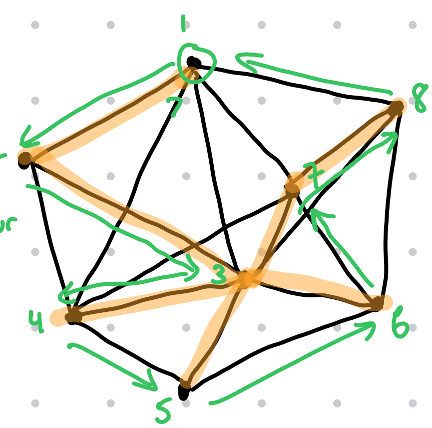
Q: How does the length of the MST relate to the length of the TSP tour? Be specific!

Let M be the length of the MST, and let T be the length of the TSP tour.

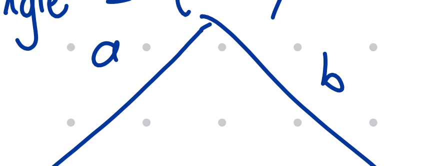
Then:

$$M < T < \underbrace{2M}$$

follow each edge of MST twice, but take shortcuts to make a TSP tour



Triangle Inequality:



$a + b > c$

LINEAR OPTIMIZATION AND THE TSP

Linear Optimization solves the following type of problem:

Find values $x_1, x_2, \ldots, x_n$

that minimize $c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$ ← linear function

subject to a set of linear equations or inequalities. } linear constraints.

examples: $x_1 + x_2 = b$

$2x_1 - 3x_2 = c$

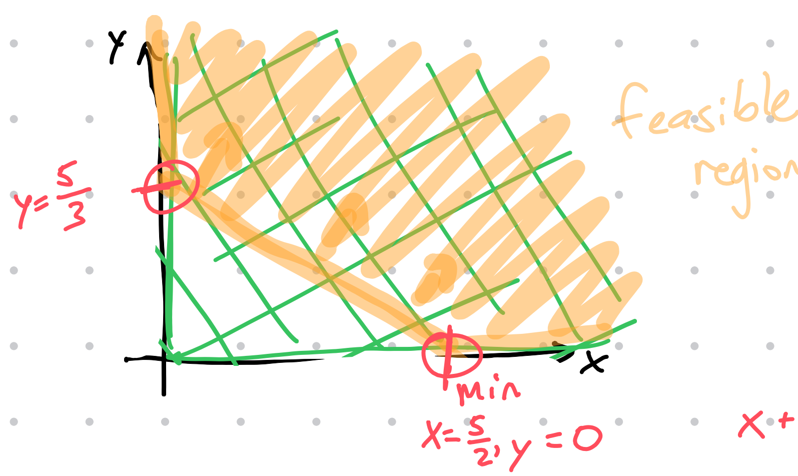
$x_1 \geq 0$

$x_2 + x_5 + x_6 - x_8 \leq 2$

Common algorithm for solving linear optimization problems is the <u>simplex algorithm</u>.

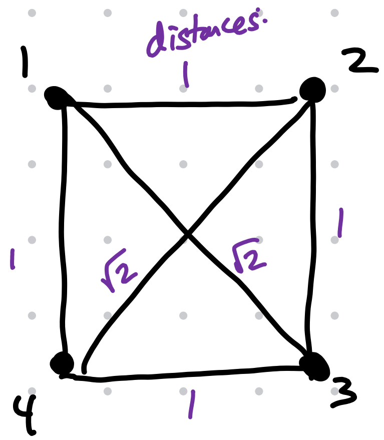
<u>Example:</u> minimize $x + 2y$ subject to

$2x + 3y \geq 5, \quad x \geq 0, \quad y \geq 0$



$x + 2y = \frac{5}{2} + 0 = \frac{5}{2}$

How do we convert the TSP to linear optimization?



VARIABLES: let $x_{ij} = 1$ if edge $i-j$ is part of the tour, and 0 otherwise

$$x_{12}, x_{13}, x_{14}, x_{23}, x_{24}, x_{34}$$

OBJECTIVE FUNCTION:

total length of tour →

$$1x_{12} + \sqrt{2}\,x_{13} + 1x_{14} + 1x_{23} + \sqrt{2}\,x_{24} + 1x_{34}$$

CONSTRAINTS:

guarantee that x_{ij} is bounded by 0 and 1

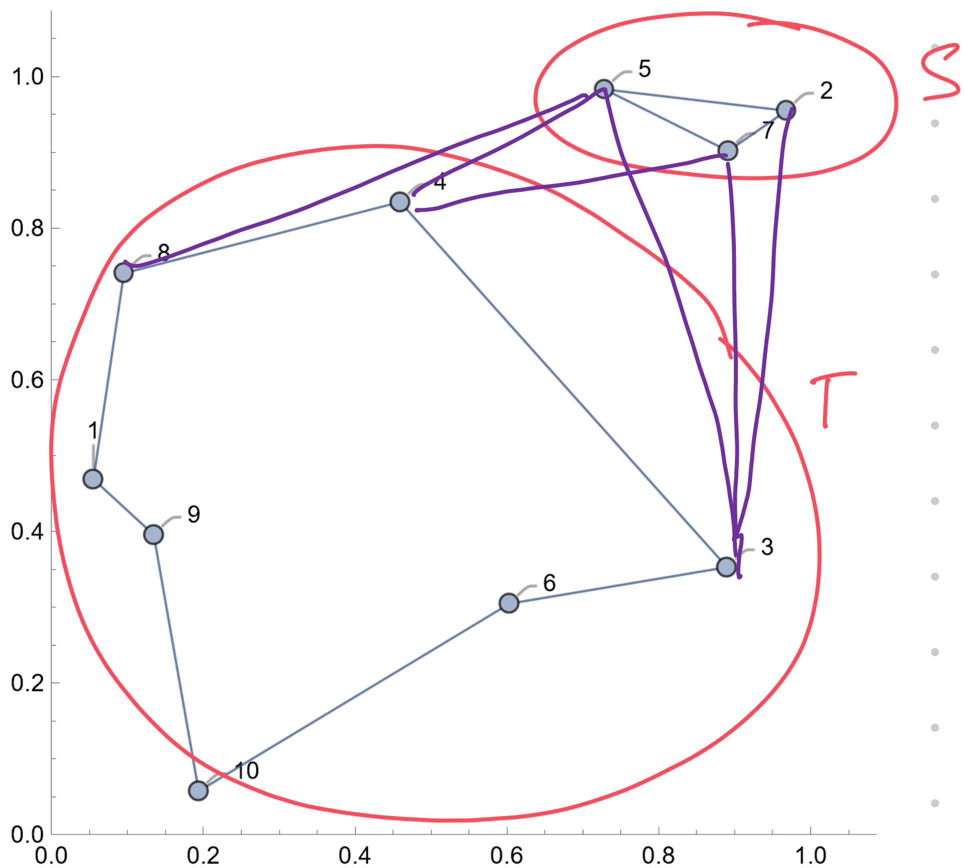
bound constraints:

$$0 \le x_{12} \le 1$$
$$0 \le x_{13} \le 1$$
$$0 \le x_{14} \le 1$$
$$0 \le x_{23} \le 1$$
$$0 \le x_{24} \le 1$$
$$0 \le x_{34} \le 1$$

guarantee that each vertex is incident to 2 edges

degree constraints:

$$x_{12} + x_{13} + x_{14} = 2$$
$$x_{12} + x_{23} + x_{24} = 2$$
$$x_{13} + x_{23} + x_{34} = 2$$
$$x_{14} + x_{24} + x_{34} = 2$$



Need at least 2 edges connecting S and T