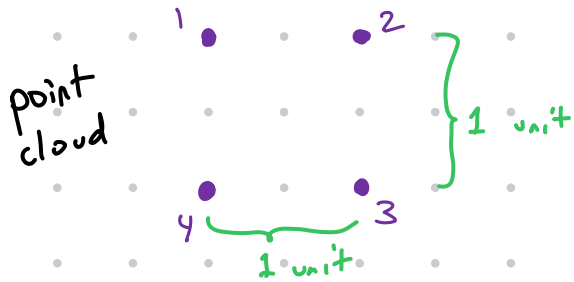
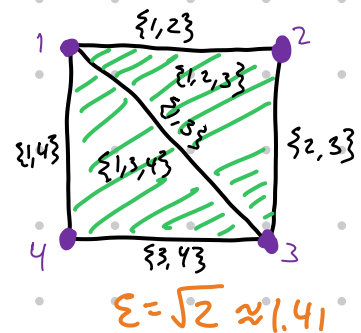
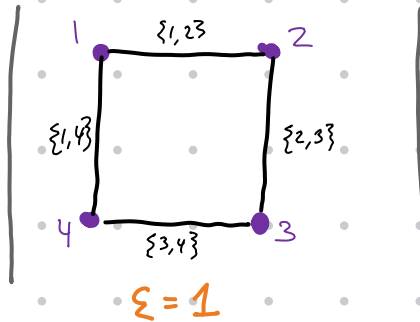
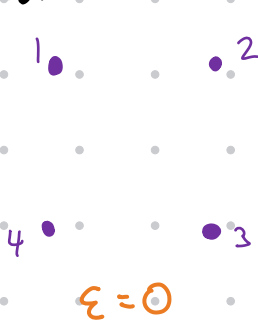


25 April 2024

PERSISTENT HOMOLOGY EXAMPLE



filtration:



boundary matrices:

$$\partial_0: \begin{array}{cccc} \{1\} & \{2\} & \{3\} & \{4\} \\ \hline & & & \end{array} \quad 0 \text{ rows}$$

4 cols

$$\partial_2: \begin{array}{cc} \sqrt{2} & \sqrt{2} \\ \{1,2,3\} & \{1,3,4\} \\ \hline \{1,2,3\} & | & | \\ \{2,3\} & | & | \\ \{3,4\} & & | \\ \{1,4\} & & | \\ \sqrt{2} \{1,3\} & | & 0 \end{array}$$

$$\partial_1: \begin{array}{ccccc} & \{1,2\} & \{2,3\} & \{3,4\} & \{1,4\} & \{1,3\} \\ \hline \{1\} & 1 & & & 0 & 0 \\ \{2\} & 1 & 1 & & 0 & 0 \\ \{3\} & & 1 & 1 & 0 & 0 \\ \{4\} & & & 1 & 0 & 0 \end{array}$$

$j=1$ $j=5$

PAIRS:

- $\{2\} - \{1,2\}$ |
- $\{3\} - \{2,3\}$ |
- $\{4\} - \{3,4\}$ |
- $\sqrt{2} \{1,3\} - \{1,2,3\}$ |
- $\{1,4\} - \{1,3,4\}$ |

UNPAIRED

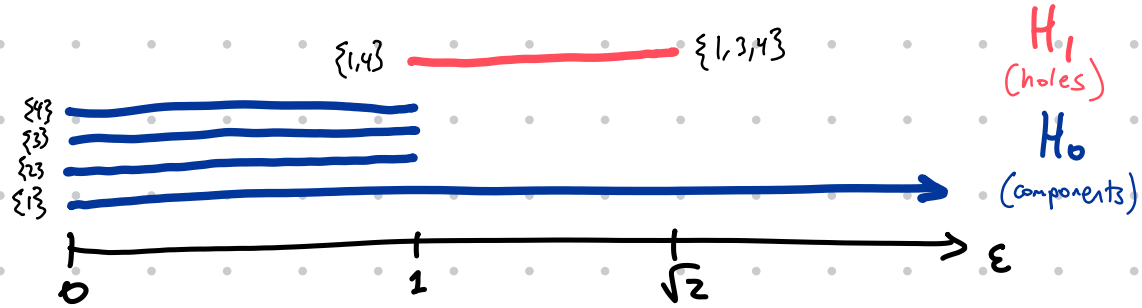
- $\{1\}$

$\sqrt{2} \{1,3\} - \{1,2,3\}$ — length 0 bar

ALGORITHM:

- For j from 1 to n :
- While there exists $j' < j$ with $\text{low}(j') = \text{low}(j) \neq 0$:
- Add col j' to col j (mod 2)

BARCODE:



PERSISTENCE PAIRS

- If $\text{low}(j) = i > 0$, then simplex j is negative, paired with simplex i
- If $\text{low}(j) = 0$, then simplex j is positive, so look to row j for pairing
- If there is no k with $\text{low}(j) = k$, then simplex j is unpaired, and thus it generates homology.